XIX. *A series of observations of the satellites of the Georgian planet, including a passage through the node of their orbits; with an introductory account of the telescopic apparatus that has been used on this occasion; and a final exposition of some calculated particulars deduced from the observations.* By William Herschel, *LL.D. F. R. S.*

Read June 8, 1815.

THE observations of the satellites of the Georgian planet, of which an account is given in this paper, are of such a nature that, in order to judge of them properly, and to make them useful to those who would continue them, it will be necessary to enter into some particulars relating to the telescopic powers required for critically viewing such difficult objects.

The great distance of the Georgian planet renders an attempt to investigate the movements of its satellites a very arduous undertaking; for their light, having to traverse a space of such vast extent before it can reach us, is so enfeebled, and their apparent diameter so diminished, that an instrument, to be prepared for viewing them, must be armed with the double power of magnifying and of penetrating into space.

With regard to the first of these requisites, I have already shown in a former Paper,* that the magnifying power of my ten feet telescope, when no uncommon degree of light is

* Phil. Trans. for 1805, page 31.

wanting, is fully equal to what may be required to view extremely small objects; but this branch of the properties of optical instruments seems not to be generally understood: the question how much a telescope magnifies, admits of various answers. To resolve it properly, we ought in all circumstances to consider how far the magnifying power of a telescope is supported by an adequate quantity of light; as without it, even the highest power and distinctness cannot be *efficient*. The question therefore ought to be limited to an inquiry into the extent of what may be called the *effective* magnifying power? It will however be found, that even then, the quantity of this power cannot be positively assigned. For if a card containing engraved letters of a certain size be put up at a given distance, the effective power of a telescope directed to it, will be that wherewith we can read these letters with the greatest facility; but if either the size of the letters, or their distance from the telescope, be changed, the quantity of this power will no longer remain the same.

An obvious consequence of this consideration is, that the effective power of telescopes has a considerable range of extent, and can only be assigned when the object to be viewed is given; and that in this determination two circumstances are concerned, which require a separate investigation; and this is abundantly confirmed when a ten feet reflector, such as has been mentioned, is directed to the Georgian planet; for with none of its highest powers can we possibly ascertain even the existence of the satellites.

Since, then, it is absolutely necessary that the power of magnifying should be accompanied with a sufficient quantity of light, to reach the satellites of this remote planet, it may be useful to cast an eye upon the action of a power which is become so essential. Its advantages and its inconveniences must equally be objects of consideration.

A very material inconvenience is that mirrors, which must be large in order to grasp much light, must also be of a great focal length; and that in consequence of this, we must submit to be incumbered with a large apparatus, which will require an assistant at the clock and writing desk, and also an additional person to work the necessary movements. The machinery of my twenty feet telescope is however so complete, that I have been able to take up the planet at an early hour in the evening, and to continue the observations of its own motion, together with that of its satellites, for seven, eight, or nine hours successively.

The forty feet telescope having more light than the twenty feet, it ought to be explained why I have not always used it in these observations. Of two reasons that may be assigned, the first relates to the apparatus and the nature of the instrument. The preparations for observing with it take up much time, which in fine astronomical nights is too precious to be wasted in mechanical arrangements. The temperature of the air for observations that must not be interrupted, is often too changeable to use an instrument that will not easily accommodate itself to the change: and since this telescope, besides the assistant at the clock and writing desk, requires moreover the attendance of two workmen to execute the necessary movements, it cannot be convenient to have every thing prepared for occasional lucid intervals between flying clouds that may chance to occur; whereas in less than ten minutes, the twenty feet telescope may be properly adjusted and directed so as to have the planet in the field of view.

In the next place I have to mention, that it has constantly been a rule with me, not to observe with a larger instrument, when a smaller would answer the intended purpose. To use a manageable apparatus saves not only time and trouble, but what is of greater consequence, a smaller instrument may comparatively be carried to a more perfect degree of action than a larger one; because a mirror of less weight and diameter may be composed of a metal which will reflect more light than that of a larger one; it will also accommodate itself sooner to a change of temperature; and when it contracts tarnish, it may with less trouble be repolished; to which may be added, that having two mirrors for the twenty feet always ready, my observations could never be interrupted by accidents which often happen to large mirrors, such as greatly injure, or even destroy their polish.

The quantity of light reflected by the mirror of a twenty feet telescope of my construction being known, and the satellites of the Georgian planet being the objects to be viewed, I may now examine the combined powers of this instrument, and assign the limits to which they may be stretched. It will however be proper first, to point out from experience some of the advantages that may be taken, if not to increase, at least not to obstruct, the penetrating power, by the full effect of which the magnifying power is to be supported.

The first precaution I ought to give is, that in these delicate observations, no double eye glass should be used, as it cannot be prudent to permit the waste of light at four surfaces, when two will collect the rays to their proper focus. The hole through which they pass in coming to the eye, should be much larger than the diameter of the optic pencils, and considerably nearer the glass than their focus; for the eye ought on no account to come into contact with the eye piece; and a little practice will soon enable the observer to keep his eye in the required situation. It is hardly necessary to add, that no hand should touch the eye piece.

With regard to the eye glasses, when merely the object of saving light is considered, I can say from experience, that concaves have greatly the advantage of convexes; and that they give also a much more distinct image than convex glasses.

This fact I established by repeated experiments about the year 1776, with a set of concave eye glasses I had prepared for the purpose, and which are still in my possession. The glasses, both double and plano-concaves, were alternately tried with convex lenses of an equal focus, and the result, for brightness and distinctness, was decidedly in favour of the concaves.

For the cause of the superior brightness and sharpness of the image which is given by these glasses, we must probably look to the circumstance of their not permitting the reflected rays to come to a focus.

Perhaps a certain mechanical effect, considerably injurious to clearness and distinctness, takes place at the focal crossing of the rays, which is admitted in convex lenses.*

* About the same time that the experiments on concave eye glasses were made, I tried also to investigate the cause of the inferiority of the convex ones; and it occurred to me, that an experiment might be made to ascertain whether the rays of light in

I have occasionally availed myself of the light of concave eye glasses, but a great objection against their constant use is, that none of the customary micrometers can be applied to them, since they do not permit the rays to form a focal image. Their very small field of view is also a considerable imperfection; in observations, however, that do not require a very extensive field, such as double stars or the satellites of *Saturn and the* Georgian planet, this inconvenience is not so material.*

As I have already shown that the *effective* power of a telescope arises from the combination of its magnifying and space penetrating powers; and have also proved that the effect of their union, when they are differently combined, must have a considerable range, it will now be easy to point out the extent of this range in the telescope by which the following observations have been made.

The magnifying power by which the satellites of the planet were discovered was only 157; but this power, which has been

crossing, jostled against each other, or were turned aside from their right lined course by inflections or deflections. With a view to this, I directed a 10 feet telescope to some finely engraved letters put up at a convenient distance. A convex eye glass was fixed to a skeleton apparatus, which left the focal point freely exposed. A concave mirror was placed so as to throw the focus of the sun's rays upon the focal image of the telescope, where, meeting with no intercepting body, they would freely pass through it at right angles. Then a screen being placed to keep off the solar rays, I fixed my attention upon the letters viewed in the telescope, and the screen being alternately withdrawn and replaced, I could perceive no sensible alteration in the brightness or distinctness of the letters. Hence I surmised, that the rays of light did not sensibly jostle in an instantaneous right angled passage, but that possibly they might suffer inflections or deflections in their crossing at the focal point on account of their being longer in collateral proximity.

* See Phil. Trans. for 1794, p. 58.

constantly used in my sweeps of the heavens, and was found to be very *effective* for the discovery of faint nebulæ and minute clusters of stars, is hardly sufficient to show the satellites steadily; for, unless every thing is favourable, their faint scintillation will only be perceived by interrupted glimpses.

The magnifiers 300, 460, 600 and 800, it will appear by the following observations, have gradually been found to be more effective on the objects on which they were used; according to the clearness of the air, the altitude of the planet, the absence of the moon, the high polish of the mirror, and other circumstances: on particular occasions, when doubtful points were to be resolved, even 1200 has been most effective. The higher magnifiers 2400, 3600 and 7200 have also been used to scrutinize the closest neighbourhood of the planet, in order to discover additional satellites; but, from the appearance of the known ones, which began to be nebulous, I concluded that these powers were not distinct enough to be used on this occasion.

As the following observations are given for the purpose of enabling astronomers to calculate the elements of the orbits and motions of the satellites with mathematical precision, I have endeavoured to save them some labour by giving a clear statement of the general outlines of them; and that some judgment may be formed of their accuracy, which I hope will be found considerable, a short detail of the method I have pursued will be necessary.

For ascertaining the position of the satellites from which their periodical revolutions were determined, three different methods have been used.

Coarse estimations were made when they seemed to be sufficient to keep the satellites in view, by way of ascertaining their identity; for unless they were followed in their course and known to be satellites, it would have been endless to measure either the distance or position of every small star that might have the appearance of one; and as the opportunities for taking measures, which require a very clear and undisturbed atmosphere, were scarce, and often interrupted by cloudy or moonlight nights, the identity of the satellites would have been doubtful if their position had not been attended to, when seen in unfavourable circumstances. When no other stars interfered, it was often sufficient barely to mention the quadrant in which they were seen, by recording that such a satellite was np, nf, sf, or sp; or if necessary, some rather more determined account, such as 40 or 50 degrees np, sf, &c.

As a check upon the description of the situations, a figure was always added to represent the planet, its satellites and the neighbouring stars as they appeared in the telescope. Very often indeed the configuration itself was deemed to be sufficient to point out the situation of the satellites, which by way of distinction were marked by numbered points; 1 and 2 being used to distinguish the known satellites; 3, 4, 5, &c. those that might possibly be other suspected, but not ascertained ones. Stars instead of points, were marked by asterisks.

More careful estimations were made with a power not less than 300, and a wire in the focus of the eye glass, to ascertain the parallel; they are capable of considerable accuracy in situations that are only a few degrees north or south preceding or following, and also when the position of a satellite is nearly 90 degrees north or south of the planet.

Measures taken with the micrometer may always be supposed to be accurate, unless they are marked as being affected by some circumstances existing at the time they were taken: when these are favourable, they can hardly be liable to any great error.

The calculations which I have given with the observations, will show the appropriate confidence each of these three methods of obtaining the positions of the satellites may separately deserve.

A much greater difficulty attaches to taking measures of distances than to those of angular positions: when the latter are taken, we have the position of the satellite in view all the time the planet passes along the parallel; and, although the moment of ascertaining the angle is only that in which the planet is in the centre of the wires, yet a constant attention to the motion of the two bodies will sufficiently enable us to perceive any excess or defect in the parallelism between their situation and that of the adjustable wire, whereas in measures of distances, the telescope must be kept in motion to retain the two bodies in their contact with the two wires, which disturbance considerably affects the delicacy of vision, and moreover requires a divided attention, as the passage of each body over its respective wire must be viewed. The only exception is, when the satellite is at 90 degrees, in which case the distance of the two bodies may indeed be measured with great accuracy.

The lucid point micrometer which has been tried is subject to the same difficulties;* its application to my construction of the 20 feet telescope, with regard to situation, is very conve-

* For a description of this micrometer, see Phil. Trans. for 1782, page 163.

nient. When the apparatus was preparing, I found that handles, 20 feet long, would be very cumbersome, and attempted to try the micrometer with the assistance of a person to arrange the points; but, when engaged in the first measure, I found that unless I had myself the command of the motions, a perfect adjustment could not be obtained; or would at least take up so much time as would bring on an alteration in the telescopic motions, not consistent with perfect vision. This micrometer has, however, the peculiar advantage that it may be used with a concave eye glass.

When a satellite is either directly preceding or following the planet, its distance may be measured by the difference of the time of their passing the meridional wire. This method, which has also been tried, is however not sufficiently delicate for very small intervals, and is moreover of little use, on account of the very limited situations.

The following observations on the satellites of the Georgian planet are given in the order of time they were made. They contain every thing that relates to the appearance and motion not only of the two principal large satellites, that are plainly within the reach of a 20 feet telescope of my construction, but also the more difficult researches that have been pursued for detecting additional satellites. That such there are I can have no doubt; but to determine their number and situation will probably require an increase of the illuminating power, such as I was in hopes, when I published my announce of their existence, would have been used by other astronomers, in pursuit of the subject pointed out to them; a 25 feet reflector which is mentioned in the observations, may probably be sufficient for the purpose.

To facilitate calculation, the observations are all given in mean time, and after each of them is added a theoretical exposition of the place of the satellites, which I have called an identification, and is denoted by the sign ‡; the great use of which will be to point out the validity of each observation, by comparing the observed places with the theoretical ones. The method of identification, which will be described hereafter, by giving not only the angle of position at which a satellite ought to have been seen, but also its proportional distance in 600 dth part of the radius of its orbit, is of great consequence when the orbits of the satellites are much contracted. These distances indeed become at last the only criterion by which we may know the satellites, for the angle of position, when the planet is near the node of the orbits, admits of so little change that it ceases to be a direction for identifying them.

The same distance will also give us the total value of the measure of any distances taken by the micrometer, so far at least as to show which of them may be the most proper to be chosen for a more rigorous investigation.

An identification of supposed satellites cannot be made by calculation; but the observations of following and also of preceding nights, accompanied by accurate configurations, may ascertain whether the object in question be of a sidereal or planetary nature. For if by the removal of the planet a supposed satellite be left in its former place, it is decidedly a star; whereas a well ascertained absence from the observed place will make its planetary nature highly probable. Then also, if a configuration and description of every small star, that is situated in, and very near the path of the planet, has been previously made, and additional stars are afterwards found to be near the planet, which cannot be accounted for, it becomes again probable that such questionable objects are of a planetary nature. And this being a kind of identification, I have added it after the calculated one, to every observation of doubtful objects, except where a supposed satellite is pointed out which there is reason to believe may be a real one; for in that case, the observations relating to the object in question, are given in their regular order.

It will not be necessary to give the configurations that were made at the time of observation; they generally contained the planet, its satellites, and some of the neighbouring stars, especially those that were in the path of the planet's motion; nor will it be necessary to mention lines and descriptions of situations of stars pointed out by letters affixed to them, as the observations are generally so redundant, that I found it highly necessary to compress them.

Observations of the satellites of the Georgian planet, accompanied by a theoretical determination of their situation, whereby their identity may be ascertained.

1787, January 11^d 12^h 13^m. There is a supposed first satellite about 42 or 43 degrees south following the planet; and a second about 45 degrees north preceding. A third supposed satellite is south following the planet.

‡ By the identifying method, it appears that a real satellite, called the first, was visible at the time mentioned about 45½ degrees south following; which agrees with the estimation of the angle of its situation, and also with a configuration of the stars and planet, drawn at the time of observation. By

the same method it appears that a real satellite called the second, was visible about 65 degrees north preceding the planet; which situation agrees with the configuration and also sufficiently well with a coarse estimation, which, as there was no wire for the parallel in the focus of the eye glass, could not be accurate. The supposed third satellite, by subsequent observations, was found to be a star.

1787, January 12. The first and second satellites are not to be seen in the place where I saw them last night. The supposed third is left where it was. I can see no small star near the planet, but the evening is not sufficiently clear.

1787, January 14, $12^h\ 3'$. There are again three supposed satellites; I have marked them 1st, 2d, and 3d, without any particular reason for that order.

‡ The first satellite was 88° nf; which agrees with the situation of that which in the configuration was marked 1st. The second satellite was $22\frac{1}{4}$ sp, and agrees with that which in the configuration was marked 3d. In the configuration the numbers are placed according to their distances from the planet, and that which is marked 2d was found to be a star remaining in its place.

1787, January 17, $11^h\ 51'$. There are now again three supposed satellites. The first is south preceding the planet, and makes a right angle with the 2d and 3d. The second is at the angular point and is south of the planet but a little preceding. The 3d is north following the planet. I have also added a 4th and 5th. The night is very fine and my telescope bears a high power.

‡ The first was 34° sp, which agrees with the configuration. The second was south of the planet but a little following,

namely $80\frac{3}{4}$° sf; which agrees sufficiently well with an estimation made without a direction for the parallel. The supposed 3d, 4th, and 5th, by next night's observation, remained in their places as small stars.

1787, January 18, $11^h\ 45'$. There are two supposed satellites; the first is directly south of the planet; the second is about 45 degrees south following, and a little farther from the planet than the first. With 480 the first is about 4 diameters of the planet distant from it; the second is about $4\frac{1}{2}$ or 5 diameters from the planet; the first is from the second about $2\frac{1}{2}$ diameters of the planet. There is no small star in the path of the planet that might be taken for a satellite to morrow.

‡ The first was $76\frac{2}{3}$ sf. The second was $59\frac{1}{3}$ sf; both these positions agree sufficiently well with the delineated configuration.

1787, January 24, $11^h\ 23'$. The first and second satellites of January 18, are no longer in the place in which they were that night. There are two satellites; the first is about 45° np the planet; the second is about 80° np; it is brighter than the first. I had a glimpse of a 3d and 4th.

‡ The first satellite was 49° np; the second was $75\frac{1}{4}$ np, which agrees well with the estimations and with the configuration. The observations of the third and fourth were lost, the planet not being seen again till eight days after, when it would have taken up too much time to look for them.

1787, February 4, $6^h\ 21'$. The first satellite is about 80° sp; the second is about 30° nf. There is too much day-light to see the satellites well. A third supposed satellite is south preceding the first; it is extremely small. There is but one single small star in the path of the planet which to morrow night may be taken for a satellite.

‡ The first satellite was 50° sp; the second was 40° nf. This differs considerably from the estimations, probably owing to the remaining day-light; the satellites however could not be mistaken, as there were no other stars near the planet.

1787, February 5, 9^h 3′. Both satellites are certainly absent from the place where I saw them last night. The first is about 85° sf; the second satellite (miscalled a small star) is by the configuration at a great angle nf. The small star in the path of the planet observed last night remains in its place.

‡ The first satellite was 89° sf; the second was 69° nf.

1787, February 6. I compared the configurations of January 11, 14, 17, 18, 24, February 4 and 5 together, and found that, admitting one of the satellites to make a revolution round the planet in about $8\frac{3}{4}$ days, and supposing its orbit to be very open to the visual ray, there was always one that would answer to a projection made on that scale.

1787, February 7, 6^h 54′. A satellite (miscalled the third) is a few degrees south following 6^h 30′, another (miscalled the first) is about 65° np. A small star (miscalled the second satellite) is about 60° sp.

‡ The first satellite was $11\frac{1}{4}$ sf. The second was $68\frac{2}{3}$ np. Two days after, the miscalled second, was seen remaining in its observed place. In the course of about nine hours of observation, I saw the planet accompanied by its two satellites, very evidently moving together in the path of the planet.

1787, February 9, 10^h 39′. Both satellites are gone from the place where I saw them the 7th of February. The first satellite (miscalled the second) is directly north of the planet; the second (miscalled the first) is a few degrees np.

‡ The first was $81\frac{2}{3}$° nf; the second 11° np.

1787, February 10, 8ʰ 57′. The first satellite is about 53° np. 8ʰ 33′. The second satellite is about 20° sp; a supposed third is about 45° sf. In a little more than four hours, I saw the satellites go on with the planet, and also in their orbits.

‡ The first satellite was $67\frac{3}{4}$° np; the second was 20° sp. The supposed third was lost, no subsequent observation having been made of it. Before I began observing, I had delineated their places on paper, on a supposition that one of them moved at the rate of $8\frac{3}{4}$, the other at that of $13\frac{1}{2}$ days the revolution.

1787, February 11, 13ʰ 28′. Between flying clouds I saw the second satellite.

‡ The satellite was $57\frac{1}{2}$° sp, which agrees with the configuration.

1787, February 13, 10ʰ 0′. The first satellite, with 300, is about 75 or 80° sp; its distance from the planet is about $\frac{3}{4}$ of a minute. The second is about 85° sf; its distance is one full minute; the estimations are by the field of view of the sweeping piece. Third, fourth, and fifth supposed satellites were marked.

‡ The first satellite was $68\frac{3}{4}$° sp; its distance was 553, the radius of its orbit being 600. The second was $80\frac{1}{2}$° sf; its distance was 599, the radius of its orbit being also 600. The third, fourth, and fifth supposed satellites proved to be stars. No great accuracy can be expected from the estimated distances given in the observation, the field of the eye piece, which took in 15 minutes, being much too large for the purpose.

1787, February 16, 9^h 38′. The two satellites are in the places where I had drawn them on paper. With a power of 300, and a wire for the parallel in the focus of the eye glass, the first satellite is, by very accurate estimation, about 5 degrees north following; at the same time, and with the same power and accuracy, the second is about 3 degrees south following. A third supposed satellite is pointed out.

‡ The position being so near the parallel, and by calculation also near the conjugate axis of the elliptical projection of the orbits, and therefore less liable to an error arising from the application of correction, have been fixed upon as standards for the calculation of the periodical revolutions of the satellites. The supposed third was next evening observed to remain in its place.

1787, February 17, 7^h 58′. I tried to measure the distance of the second satellite from the planet by a lamp micrometer. The lucid points were 246,4 inches from the eye, and when they were 14,4 inches from each other, I found that the adjustment of the distance, and angle of position could not be made to my satisfaction by an assistant, and gave up the measure. The magnifying power being 157, the opening of the points gives the angular distance 1′ 17″; but the measure when given up was still much too large.

‡ The satellite was $28\frac{1}{2}$° nf, and the distance 505.

1787, February 19, 7^h 55′. Having delineated the situation of the satellites on paper, I found them in the expected situation. A third and fourth were added in the configuration.

‡ The first satellite was 58° np; the second was 82° nf. The supposed third and fourth proved to be stars.

1787, February 22, 7^h 14′. By the configuration the first satellite is at a considerable angle sp; the second is at a moderate angle np. Third, fourth, and fifth satellites were noticed.

‡ The first was 76° sp; the second was 31° np. A long interval happening to prevent subsequent observations, the supposed satellites were lost.

1787, March 5, 7^h 14′. The first satellite is about 6° sf. 7^h 17′, the second is about 87° nf; a third is about 40° nf.

‡ The first satellite was $20\frac{1}{4}$° sf; the second was 87° np; the third proved to be a star. The planet was only observed about 3 or 4′, and it does not appear that great accuracy in the estimations was attempted.

1787, March 7, 7^h 12′. The first satellite is 82° np. 7^h 13′ the second is about 30° np. Very coarsely estimated. A third is about 6° nf; it seems to have a fourth close to it. Having some doubts about the fourth, I viewed it with 600 and 800; I saw it also well with 1200, and had a glimpse of it with 2400. These high magnifiers require a fine apparatus for adjusting the focus.

‡ The first satellite was $78\frac{3}{4}$° nf; the second was 41° np; the third and fourth proved to be stars.

1787, March 8, 8^h 52′. Both satellites were seen for a few minutes.

‡ The first was 70° np; the second was $9\frac{1}{2}$° np.

1787, March 11. I found that some friends who came to view the satellites saw them best with 480, when the planet was drawn to the margin just out of the field.

1787, March 15, 8^h 7′. The first satellite is about 48° nf; the second is 5° sf. The second satellite being so nearly following the planet, I tried to measure its distance by sidereal time. Of eight transits, four gave 3″; three gave 3″,5; and one gave 4″; a mean of them is 3″,31; and the declination of the planet being 21° 57′ north, we have the apparent distance 45″,99; but I do not trust much to measures by time, in the manner these were taken without a system of wires in the focus of the eye glass, and with the clock and assistant at a considerable distance.

‡ The first satellite was 46° nf; the second was 2¼° sf, and its distance from the planet was 481; this would give the greatest elongation 57″,36 which is probably much too large.

1787, March 18, 8^h 3′. The satellites are in the place where I expected them. The first is 5 or 6° np; the second is about 75° nf; it seems to be farther from the planet than when it was near the parallel. I attempted to measure its distance by the parallel wire micrometer; eclipsing the satellite with one wire, and bisecting the planet with the other. The measure gave the distance 46″,46.

‡ The first was 21° np; the second was 83½° nf; and its distance was 588; which gives the greatest elongation 47″,41.

1787, March 19. Both the satellites are in their expected situation, which for the first is 36° sp; for the second 79° np. At 7^h 48′ I took a good measure of the distance of the second satellite; it gave 44″,24. I attempted a second measure, but was interrupted before I had quite finished it to my liking; it gave 45″,98.

‡ The first satellite was 29¾° sp; the second was 74⅓° np. and its distance 596. The expected situations, though calculated from imperfect tables, were sufficient to show that the satellites were not mistaken.

1787, March 20, 7^h 44′. I took three measures of the distance of the second satellite; the first gave 40″,23: the second, with the remark, pretty full measure, gave 41″,89; the third with the addition, not too large, gave 40″,20.

‡ The satellite wa $52\frac{2}{3}$° np, and the distance 564.

1787, April 9, 10^h 22′. I took two very accurate measures of the distance of the second satellite from the planet; the first gave 44″,54, the second 44″,35. By temporary tables its expected place was 57° sf.

‡ The satellite was 54° sf; and its distance 563.

1787, April 11, 9^h 18′. By temporary tables the expected situation of the second satellite was 4° sf. I took three good measures of its distance from the planet; the first gave 34″,47; the second 35″32; the third 35″,74. A mean of them is 34,″99.

‡ The satellite was $1\frac{1}{2}$° nf; and its distance 477.

1787, April 17, 8^h 53′. The two satellites are on opposite sides of the planet.

‡ The first was $40\frac{1}{2}$° sf; distance 531. 9^h 6′, the second was $21\frac{2}{3}$° np; distance 503.

1787, September 19, 15^h 55′. The first satellite (miscalled second) is 85° sp; the second (miscalled first) is about 30° sf.

‡ The first was $87\frac{3}{4}$° sp; the second was $10\frac{3}{4}$° sf. This being the first time of seeing the planet after its conjunction, accounts for the mistakes of the names.

1787, October 11, 16^h 49′. The first satellite (miscalled second) is 78° np. Two good measures of its distance from the planet were taken; the first gave 35″,18, the second 35″,96; a mean is 35″,57. 16^h 51′. The second satellite (miscalled first) is 40° sp.

‡ The first was 84° np, and its distance 599; the second was $52\frac{2}{3}$° sp; and its distance was 492. The long interruption in the observations was again the cause of a mistake of the names, which the calculation sets right.

1787, October 14, 15^h 59′. The angle of position of the first satellite by the micrometer is 48° 22′ sp; that of the second at 16^h 29′ is 66° 2′ sf.

‡ The first satellite was $49\frac{1}{2}$° sp; the second was $65\frac{1}{2}$° sf.

1787, October 20, 15^h 36′. Position of the first satellite by the micrometer 72° 0′ np. Position of the second at 16^h 8′, 80° 12′ np.

‡ The first was $76\frac{2}{3}$° np; the second was $80\frac{1}{2}$° np.

1787, November 9, 15^h 56. The second satellite is about 87° sf. The distance by four good measures 46″,15; 43″,92; 42″,94; 46″,57; mean 44″,89.

‡ The satellite was 84° sf; distance 594.

1788, January 14, 12^h 3′. The two satellites are almost in opposition; but the first precedes a line continued from the second through the planet.

‡ The first satellite was $77\frac{1}{4}$° nf; the second was 66° sp.

1789, February 22, 9^h 48′. The first satellite is about 80° sp; the second is about 85° sp; too much wind for measuring.

‡ The first satellite was $69\frac{1}{2}$° sp; the second was $85\frac{1}{3}$° sp.

1789, February 24, 9^h 13′. The first satellite is a few degrees more advanced in its orbit than the second.

‡ The first satellite was $48\frac{1}{3}$° sf; the second was 55° sf.

1789, March 13, 9^h 1′. The first satellite is 60° sf. 7^h 47′, the second is about 45° nf, third and fourth satellites were marked.

‡ The first satellite was $63\frac{1}{3}°$ sf; the second was $57\frac{2}{3}°$ nf. The third and fourth were found to be stars.

1789, March 14, 9^h 22′. The first satellite is 8° sf; the second is 70° nf.

‡ The first satellite was $19\frac{1}{4}°$ sf; the second was $81\frac{1}{4}°$ nf.

1789, March 16, 7^h 33′. The first satellite is 83° nf; the second is about 60° np. A third, about 2° sf; a fourth, about 8 or 10° np.

‡ The first satellite was $73\frac{1}{3}°$ nf; the second was 61° np. The third and fourth were stars.

1789, March 20, 7^h 50′. The two satellites were coarsely estimated to be at considerable angles sp.

‡ The first satellite was 63° sp; the second was $65\frac{1}{3}°$ sp.

1789, March 26, 10^h 44′. A star was mistaken for the first satellite; the second satellite (miscalled the first) is 45° nf.

‡ The first satellite was $63\frac{1}{4}°$ np; the second was 50° nf.

1789, December 15, 10^h 54′. The first satellite is about 71° sp. 10^h 49′, the second is about 75° sp; a third is about 75° sf.

‡ The first satellite was 72° sp; the second was $81\frac{1}{3}°$ sp; the third was a star.

1789, December 16, 10^h 12′. The first satellite is about 83 or 84° sf; the second is 85° sf. By the configuration they are very nearly in conjunction.

‡ The first was $83\frac{2}{3}°$ sf; the second was 83° sf.

1790, January 18, 9^h 32′. The first and second satellites are in the places I had calculated. There is a supposed third satellite about two diameters of the planet following, extremely faint and only seen by glimpses; 1^h 6′ after I could not perceive it; a fourth is about 70° np.

‡ The first was $38\frac{3}{4}$° sp; the second $85\frac{1}{4}$° nf.

1790, January 19, 9^h 34′. There is a very small star left in the place where the supposed fourth satellite was last night. 10^h 47′, I can see no fourth satellite near the second where it would be now if it had been a real satellite. With the assistance of a field bar to hide the planet; and a power of 300, I can see the first and second satellites very steadily, even the very first moment I look into the telescope.

‡ The first satellite was $76\frac{2}{3}$° sp; the second was $77\frac{1}{4}$° np. It is very strange that the third supposed satellite should not have been attended to when two observations are given to prove that the supposed fourth was not a satellite.

1790, January 20, 12^h 5′. The first and second satellites are in the places I had calculated; a third satellite is 45° np, and in a line with the planet and the second satellite.

‡ The first satellite was $77\frac{1}{4}$° sf; the second was $54\frac{1}{2}$° np. The third was not accounted for.

1790, February 6, 9^h 28′. I viewed the planet and satellites with three concave eye glasses, power about 240, 320, and 460. I see very clearly with these glasses. Cloudy.

‡ The first was $89\frac{3}{4}$° sf; the second was 64° sp.

1790, February 9, 9^h 19′. By a configuration the first satellite is at a considerable angle nf; the second at a great angle sf. A third is in a line with the planet and the second satellite; its distance from the planet by the configuration is about twice that of the second satellite.

‡ The first was $48\frac{3}{4}$° nf; the second was $61\frac{1}{2}$° sf; the third was $61\frac{1}{2}$° sf; two succeeding observations are decisive that

the supposed third satellite was not a star remaining in its place.

1790, February 11, 8° 30′. The satellites are in the places I had calculated. 8^h 56′, the small star of the 9th of February I believe is wanting; at least I cannot see it though the weather is very clear, but windy. An additional third and fourth are pointed out.

‡ The first satellite was 74° np; the second was 7° nf. The third and fourth of this night were found to be stars.

1790, February 12, 11^h 27′. The first and second satellites are in the places I had calculated. The third and fourth of last night are small fixed stars remaining in their places. The supposed third satellite of the ninth is not in the place where I saw it that night.

‡ The first satellite was 27° np; the second was $48\frac{1}{3}$° nf.

1790, February 16, 8^h 2′. The first and second satellites are in the places I had calculated; the situation of a supposed third is described.

‡ The second was $56\frac{1}{3}$° np; the supposed third proved to be a star.

1790, February 17. A configuration of stars situated in the planet's path is delineated.

1790, March 3, 7^h 58′. The first satellite is 40° sp. 8^h 42′, the second is 3 or 4° np.

‡ The first satellite was 56° sp; the second was $0\frac{1}{3}$° np.

1790, March 5, 10^h 38′. The first and second satellites are in the places I had calculated; a 3d, with 600 is 56° sf; a fourth is delineated.

‡ The first satellite was 63° sf; the second was 66° sp; the third and fourth proved to be stars.

1790, March 8, 10^h 43′. Forty feet telescope. I saw the satellites with great ease. The speculum being extremely tarnished, I did not expect to have seen so well as I did.* Twenty feet telescope. The first satellite is 85° 7′ nf. 8^h 39′, the second is 67° 36′ sf. My wire is too fine and the power 460 too high for great accuracy.

‡ The first satellite was $79\frac{1}{3}$° nf; the second was $59\frac{1}{2}$° sf.

1790, April 3, 9^h 39′. The first satellite is on the opposite side of the second; the position of the second is 77° 53′ sf.

‡ The first satellite was 75° nf; the second was $76\frac{2}{3}$° sf.

1791, January 31, 11^h 5′. The second satellite is 74 or 75° np. A supposed satellite in opposition to the second, and at double its distance from the planet, is marked in the configuration.

‡ The first satellite was $0\frac{3}{4}$° nf; its distance was 336, and not being noticed it was probably invisible; the second was $78\frac{3}{4}$° np; the supposed exterior satellite was $78\frac{3}{4}$ np.

1791, February 2, 8^h 23′. The first satellite is about 70° nf. 8^h 10′, the second is gone with the planet from the stars of the configuration of the 31st of January.

‡ The first satellite was 81° nf; the second was 30° np. The lettered stars of the configuration were all named as being left in their places, but the supposed exterior satellite of that day is not mentioned among them.

1791. February 4, 8^h 13′. The second satellite is 40° 48′ sp, but the measure is imperfect and may be out 5 or 6 degrees; a supposed satellite was marked, and a small star pointed out in the path of the planet.

* See Phil. Trans. for 1814, p. 275. A note relating to the polish of the 40 feet mirror.

‡ The second satellite was 52° sp. The supposed satellite was found to be a star.

1791, February 5, 11^h 5′. The first satellite is 20° sp. 10^h 45′, the second is 65° sp. With 600, third, fourth and fifth satellites are marked: but as they are also visible with 300, they are probably stars.

‡ The first satellite was 41° sp; the second was $74\frac{1}{2}$° sp. The third, fourth and fifth, were lost for want of subsequent observations.

1791, February 22, 8^h 23′. I cannot perceive the first satellite, probably owing to its nearness to the planet; I am pretty sure the orbits are contracted, so that the planet is approaching towards their node. 7^h 30′, a measure of the position of the second satellite is 36° 18′ sf.

‡ The first satellite was $8\frac{3}{4}$° sp; distance 333, which may account for its not having been seen. The second was 39° sf.

1791, February 23, 7^h 59′. Position of the first satellite 56° 33′ sp.

‡ The first was $60\frac{1}{2}$° sp; the second was $7\frac{1}{2}$° nf; the distance was 331, at which the satellite is sometimes invisible.

1791, March 1, 11^h 47′. The two satellites are in the places I had calculated.

‡ The first was $73\frac{1}{3}$° np; the second was $20\frac{3}{4}$° np.

1791, March 2, 9^h 18′. The first satellite is hardly to be seen; it seems to be in about the most contracted part of its orbit; the second is only about two diameters of the planet from the edge of the disk, but the estimation cannot be very accurate, as I am obliged to hide the planet to see the satellite.

‡ The first was $37\frac{1}{3}$° np; its distance 377. The second was $22\frac{1}{2}$° sp; its distance 358.

1791, March 5, 7^h 56′. The first satellite is about 75° sp; the second about 85° sp; third, fourth, and fifth satellites were pointed out.

‡ The first satellite was 87° sf; the second was $89\frac{1}{2}$° sf; the third, fourth and fifth proved to be large stars, the nearness of the planet having diminished their lustre when observed as satellites.

1791, March 6, 12^h 2′. The first satellite is much nearer the planet than it was last night; the second is also nearer, but not much.

‡ The first satellite was $50\frac{1}{2}$° sf; the second was $70\frac{1}{2}$ sf.

1791, March 9, 9^h 52′. The first satellite, with a wire for the parallel and 300, is about 86° nf. The position being so near the perpendicular cannot be much out. By the micrometer it is 86° 25′ nf. 9^h 36′, the second is on the following side; it is nearer the planet than the first, and on that account appears smaller.

‡ The first satellite was $86\frac{3}{4}$° nf; distance 599; the second was $32\frac{1}{2}$° nf; distance 379.

1791, April 4, 8^h 43′. The first satellite is 84° 56′ nf; the second was not observed.

‡ The first was $82\frac{3}{4}$° nf; the second was $9\frac{3}{4}$° sf; its distance being 343 it might not be visible.

1791, December 19, 11^h 45′. I do not perceive the first satellite; the second is about 75° nf.

‡ The first was $7\frac{1}{3}$° sf, and its distance being 263, it was therefore invisible. The second was $82\frac{1}{2}$° nf.

1792, January 27, 11^h 58′. The first satellite was not observed; the second is about 40° nf, the estimation may be out 6 or 8 degrees. Cloudy.

‡ The first satellite was $20\frac{1}{3}°$ np; its distance being 283 it could not be seen. The second was $62\frac{1}{4}°$ nf.

1792, February 12, 8^h 28′. The first and second satellites are in the same line, and I measured their position together, it is 88° 19′ np; a supposed third is 84° 23′ sp.

‡ The first satellite was $86\frac{1}{3}°$ np, and its distance 576; the second satellite was $83\frac{3}{4}°$ np; the third proved to be a star.

1792, February 13, 8^h 42′. Forty feet reflector, with 360, I saw the disk of the planet very well defined. Twenty feet. The satellites are advanced in their orbits; the first is drawn much nearer to the planet than it was yesterday; a very small star is 41° 22′ nf.

‡ The first satellite was $56\frac{1}{2}°$ np; its distance 389; the second was $66\frac{1}{3}°$ np. The very small star was left in its place.

1792, February 20, 12^h 57′. The first satellite is 89° 58′ nf; 13^h 8′, the second is 53° 21′ sf; a supposed third is 66° 17′ np.

‡ The first satellite was $83\frac{1}{3}°$ nf; the second was $53\frac{1}{3}°$ sf. By an increase of 25 or 30° in the angle of the third it was the same evening proved to be a star.

1792, February 21, 9^h 10′. Position of the first satellite 73° 52′ np.—9^h 30′, I suspected the second satellite to be in its calculated place, but even 600 would not verify it.

‡ The first satellite was $78\frac{2}{3}°$ np; the second was $17\frac{1}{3}°$ sf; distance 292.

1792, February 26, 11^h 30′. The position of the first satellite is 42° 49′ sf.—8^h 2′, that of the second is 73° 49′ np. A very small star between the planet and the second satellite is pointed out, and another towards the south at double the distance of the first is marked in the configuration.

‡ The first satellite was $52\frac{3}{4}$° sf; the second was $75\frac{1}{3}$° np. The small star was left in its place; but the distant one is not accounted for.

1792, February 28, 10^h 52′. The position of the first satellite is 69° 43′ nf. The second was not seen.

‡ The first satellite was $65\frac{1}{2}$° nf; the second was $3\frac{1}{2}$° sp; distance 293, and therefore invisible.

1792, March 15, 10^h 3′. I cannot see the first satellite with 300, 480, nor with 600. The second satellite is 73° 22′ sp.

‡ The first satellite was 17° sf; its distance was 302 and therefore invisible; the second was $74\frac{1}{2}$ sp.

1792, March 18, 8^h 19′. The first satellite is 82° 35′ np. 8^h 37′, the second is 60° 16′ sf.

‡ The first satellite was $81\frac{3}{4}$° np; the second was 56° sf.

1792, March 19, 8^h 20′. The first satellite is 38° 4′ np, I see it very well notwithstanding it is near the planet. 8^h 42′, I cannot see the second with 300. With 480 I see it very well; I see it also with 800 and 1200; I tried 2400 and 4800, but a whitish haziness in the air prevents my seeing it with these powers.

‡ The first was 46° np and its distance 364; the second was 15° sf; its distance 299 which accounts for the difficulty of seeing it.

1792, March 23, 8^h 21′. I see the first satellite through flying clouds; the second is 89° 21′ np.

‡ The first satellite was $61\frac{2}{3}$° sf; distance 430. The second was $88\frac{2}{3}$° np.

1792, March 27, 11^h 6′. The first satellite (miscalled a very small star) is about 80° np; the second s by the configuration about 45° sp; a third (miscalled the first) was pointed out.

‡ The first satellite was 70° np; the second was 46½° sp; the third was not accounted for.

1792, March 30, 11ʰ 18′. The first satellite by the configuration is at a great angle sp; the second is at a great angle sf.

‡ The first satellite was 78½° sp; the second was 83° sf. The first satellite (miscalled a star the 27th) is gone from the place where it was.

1793, February 5, 9° 18′. Neither the first nor second satellites are visible. A very small star is 19° 3′ sp.

‡ The first satellite was 30° sp; distance 282; the second was 23° sf; distance 2 45. There is no subsequent observation of the small star.

1793, February 7, 9ʰ 38′. The first satellite is 79° 39′ sp. 9ʰ 20′, the second is 59° 51′ nf. The wind being very troublesome, the measures cannot be very accurate: The difficulty was in finding the parallel.* I viewed the planet with 240, 320, 480, 600, 800 and 1200, but saw no satellites nearer than the two known ones. The north following satellite being farther from the planet than the south following one, I take it to be the second; the difference of their distance appeared plainest with 1200. I viewed the planet also with 2400, 3600, 7200.

‡ The first satellite was 86¾° sp. Distance 590. The second was 68¼° nf; distance 513; and supposing the radius of the orbit of the first to be to that of the second as 3 to 4, we have the apparent distance of the first to that of the second as 177 to 205.

* Telescopic vision in windy weather is generally very perfect, and except in cases which require an uninterrupted steadiness of the instrument, will admit of the highest magnifying powers.

1793, March 8, 11^h 21′. The first satellite is about 65° nf; the second is 90° nf; a third is about 75° nf.

‡ The first was $57\frac{3}{4}$° nf; the second was $89\frac{2}{3}$° nf; the third was a star.

1793, March 9, 10^h 35′. The first satellite is 85° nf; the second is about 82° np; a third is about 65° sp.

‡ The first was 80° nf; the second $77\frac{1}{2}$° np; there is no subsequent observation of the third.

1793, March 14, 9^h 37′. The first and second satellites are seen in their places; the situation of a third and fourth is pointed out. The first satellite is brighter than the second.

‡ The first was $89\frac{2}{3}$° sf, and 16° 26′ from its greatest elongation; distance 574. The second was 80° sp, and 7° 56′ from its greatest elongation; distance 588. The superior brightness of the first therefore could not arise from its greater distance. The third and fourth supposed satellites had no subsequent observations.

1793, April 3, 10^h 53′. The first satellite is 50° nf; the second is 80° nf.

‡ The first was $52\frac{2}{3}$° nf; the second was 79° nf.

1794, February 21, 8^h 24′. The first satellite is about 88° nf; the second is about 86° nf.

‡ The first was 89° nf; the second 85° nf.

1794, February 25, 8^h 24′. By a configuration the first satellite is at a great angle sp; and its distance from the planet is greater than that of the second, which is at a much smaller angle sp. Several small stars are pointed out.

‡ The first satellite was 84° sp; distance 593. The second was $47\frac{3}{4}$° sp; distance 323. The stars were left in their places.

February 26, 8^h 28′. The first satellite is 70° 53′ sf. 8^h 7′, the second is 66° 56′ sp; many small stars are pointed out.

‡ The first satellite was 78° sf; the second was $67\frac{1}{3}$° sp; the stars remained in their places.

1794, February 28, 9^h 43′. The first satellite is 62° 55′ nf. 9^h 26′, the second is 86° 44′ sp. 8^h 15′, there is a very small star which I did not see the 26th; it is brighter than a lettered star not far from it. Its position is pointed out by the stars of the configuration.

‡ The first satellite was $61\frac{1}{2}$ nf; the second was $87\frac{1}{2}$° sp. The stars remained in their places. The position of the small star of the 26th, by identification was about 24° nf.

1794, March 2, 8° 25′. The first satellite seems to be at its greatest elongation; the second satellite was not seen.

‡ The first was 86° np; distance 507; the second was $55\frac{3}{4}$° sf; distance 275, therefore not visible.

1794, March 4, 11^h 22′. I can see neither the first nor the second satellite. A third satellite is 61° 32′ nf. Many small stars are pointed out.

‡ The second satellite was $57\frac{1}{2}$° nf; its distance 383; it was therefore visible, and its position agrees with the measure taken of a satellite miscalled the third: the inaccuracy of my tables in 1794, occasioned the mistake. The small stars remained.

1794, March 5, 11^h 10′. The first satellite is 75° 50′ sp. 10^h 57′, the second is 72° 27′ np. There is no star in the place where the supposed third was last night. Many small stars are again pointed out.

‡ The first was 76° sp; the second was $72\frac{1}{4}$° nf. The absence of the miscalled third confirms the mistake, and

is a proof of the great attention that was paid to ascertain the nature of supposed satellites. The small stars remained.

1794, March 7, 11^h 18′. I cannot perceive the first satellite. 10^h 57′, the second is nearer the planet than it was the last time I saw it. Small stars are pointed out.

‡ The first was 63° sf; distance 320, invisible. The second was $87\frac{1}{3}$° np. There is no subsequent observation of the small stars.

1794. March 17, 7^h 38′. I can see neither of the two satellites.

‡ The first was 41° nf; distance 292, invisible. The second was $38\frac{2}{3}$° nf; distance 285, invisible.

1794, March 21, 11^h 53′. I cannot see the first satellite. I looked at several different hours for it. 10^h 53′, the second satellite is 88° 8′ np. 9^h 19′, the place of a small suspected star is pointed out, but it cannot be verified with 460 and 600.

‡ The first satellite was 27° sp; distance 243, invisible. The second was $81\frac{1}{2}$° np. The suspected star was seen in its place the following night.

1794, March 22, 8^h 47′. There is no mention of the first satellite; the second is 61° 46′ np.

‡ The first was $66\frac{1}{4}$° sp, and its distance being 480, it must have been seen and taken for a star. The second satellite was $60\frac{3}{4}$° np; distance 311.

1794, March 23, 8^h 32′. The first satellite is one of two small stars that are south of the planet; it is the preceding and largest of the the two. 8^h 42′, the second satellite is not visible.

‡ The first was 82° sp, 1° 26′ past its greatest elongation The second was 1° np, 1° 49′ past its shortest elongation, distance 207; invisible.

1794, March 26, 9^h 2′. Position of the first satellite 61° 53′ nf, as accurate as the faintness of the satellite will permit. 8^h 48′, the second satellite is 77° 0′ sp. very accurate. 9^h 17′, I suspect a third satellite directly north a little farther from the planet than the first, and the power 480 almost verifies the suspicion. 9^h 26′, with 600 I still suspect the same, but cannot satisfy myself of the reality. 11^h 24′, I see the supposed third satellite perfectly well now; it is much smaller than the first, and in a line with the planet and the first. An extensive configuration is delineated.

‡ The first satellite was $56\frac{2}{3}$° nf; the second was $79\frac{1}{2}$° sp; the third satellite being in the position of the first at 11^h 14′, must have been $59\frac{1}{2}$° nf.

1794, March 27, 10^h 25′. The first satellite is 75° 59′ nf. 10^h 12′, the second is 88° 35′ sf. 8^h 15′, the small star observed last night at 11^h 14′ is gone from the place where I saw it. From its light last night compared to a star marked *r* in the configuration which to night is very near the planet, and scarcely visible, I am certain that it must be bright enough to be perceived immediately, if it were in the place pointed out by the configuration. 11^h 19′, I have many glimpses of small stars, one of them is in a place a little north following the first satellite, agreeing with what would probably be the situation of the third satellite of last night if it had moved with the planet. A supposed third of this evening is preceding the first satellite, but nearer the planet. A supposed fourth is sf; its distance is almost double that of the second satellite. Some other small stars or supposed satellites are seen to the south at a good distance.

‡ The first satellite was 79° nf; the second was 81¼° sf. The fourth proved to be a star.

1794, March 28, 9^h 12′. The first satellite is 88° 31′ np. 9^h 1′, the second is 82° 7′ sf.

‡ The first satellite was 87° np; the second was 78° sf.

1794, April 1, 9^h 14′. The first satellite i 83° 2′ sp. 9^h 23′, the second is 70° 26′ nf.

‡ The first satellite was 87¼° sp; the second was 72° nf.

1796, March 4, 11^h 10′. The Georgian planet is about 13′ of space preceding and 5 or 6′ north of a nebula. An extensive configuration was made, but no satellites were noticed.

‡ The nebula was No. 272 of the first class of my third catalogue. (See Phil. Trans. for 1802, page 505.) The first satellite was 42⅓° sp; distance 158, invisible. The second was 87½° np, distance 364, and after an interval of near two years might possibly be overlooked. The stars pointed out remained in their places.

1796, March 5, 12^h 25′. The first satellite is 72° 20′ sp. 10^h 53′, I suspect a very small star between two of the lettered stars of last night's configuration which at the time it was made was not there. I had a pretty certain glimpse of it.

‡ The first satellite was 75° sp.—10^h 25′, the second was 72° np; distance 123, invisible. By the configuration the suspected star was at a considerable distance, about 72° np.

1796, March 9, 11^h 35′. The first satellite is 70° 36′ nf. 11^h 22′, the second is 83° 35′ sp. As the probability is that other supposed satellites move in the same plane with the first and second, I chiefly look for them in the direction of the position of their orbits which is now nearly a straight line; a star that may possibly be a distant satellite is pointed out.

‡ The first satellite was $70\frac{2}{3}$° nf. The second was 81° sp; the star remained in its place.

1796, March 10, 11^h 43′. The first and second satellites by a configuration, are not far from being in opposition, the first not being come to a line drawn from the second through the planet. With 600 there is no star between the satellites and the planet that may be supposed to be an inner satellite; with this power the satellites are very large and visible, I see them better than with a lower.

‡ The first satellite was $79\frac{3}{4}$° nf; the second was $86\frac{1}{2}$° sp.

1796, March 27, 10^h 6′. The first satellite is in the place I had calculated.

‡ It was $75\frac{1}{2}$° nf.

1796, March 28, 10^h 7′. The first and second satellites are in the places I had calculated; the apparent contraction of their orbits is such as to approach to a straight line.

‡ The first satellite was 84° nf; the second was 74° nf.

1796, April 4, 11^h 16′. The first satellite is not visible; the second is near a small sp. star. There is no star in the transverse of the apparent elliptical orbit that could be taken for a satellite, unless that near the second should be one going towards its greatest elongation, or coming from it.

‡ The first satellite was 68° nf; its distance 415; how it happened not to be visible I cannot account for; the configuration has no star near the place; the second satellite was $76\frac{1}{3}$° sp. The star near the second remained in it former situation.

1796, April 5, 10^h 48′. The first and second satellites are apparently in opposition, the same wire covers them both and the planet. There is no star in the line of the transverse that can be taken for a satellite: the night being beautiful, I examined that line with 300 at a distance, and with 600 within the orbit of the two satellites.

‡ The first satellite was $74\frac{2}{3}°$ nf; the second was $81\frac{2}{3}°$ sp.

1797, March 15, 9^h 44′. The first and second satellites are not far from their opposition; by the configuration they are short of it.

‡ The first was $78\frac{3}{4}°$ sp; the second was $81\frac{1}{4}°$ nf.

1797, March 17, 9^h 51′. The first and second satellites are both invisible; the night is very beautiful and I have a field bar to hide the planet; but notwithstanding this, I cannot see either of the satellites; many stars are pointed out.

‡ The first was $69\frac{3}{4}°$ sf; distance 110, invisible. The second was 76° np; distance 126, invisible. The stars had no subsequent observation.

1797, March 21, 10^h 9′. The first satellite is not visible; the second is nearly at its greatest elongation, it is about 70° south preceding; many stars are pointed out.

‡ The first satellite was $88\frac{1}{2}°$ np; distance 240, invisible; the second was $79\frac{1}{2}°$ sp; distance 588. The stars remained.

1797, March 23, 10^h 28′. With 320, I see neither of the satellites. 10^h 32′, having just been told where the second should be, I perceived it in its place; with 600 I see it very well; many stars are pointed out.

‡ The first satellite was 76° sp; distance 518; it does not appear why it could not be seen. The second was 88° sp; distance 298. The stars remained.

1797, March 25, 10^h 40′. With 320 I cannot perceive either of the satellites; with 600 I can see neither of them; many stars are pointed out.

‡ The first satellite was 87° sp; distance 320, invisible. The second was $66\frac{2}{3}$° nf; distance 251, invisible. The stars remained.

1797, March 28, 10^h 36′. The two satellites seem to be nearly in a line drawn from the second to the centre of the planet; the second is 80° 26′ nf. There is an exceeding small star about four times the distance of the second satellite in the line of the greatest elongation, I do not remember to have seen it among the lettered stars which are pointed out, the 25th.

‡ The first satellite was $78\frac{1}{2}$° nf; distance 597. The second was $80\frac{1}{3}$° nf; distance 592. There is no subsequent observation of the small star.

1797, March 29, 9^h 39′. I see one of the satellites. Cloudy. I suppose it to have been the second.

‡ The first was 83° nf; the second was $83\frac{1}{3}$° nf.

1798, February 6, 11^h 29′. The second satellite (miscalled a star) is at a great angle sp; three stars are pointed out.

‡ The first satellite was $74\frac{1}{3}$° nf; distance 251; invisible; The second was 79° sp; distance 552. The three stars remained.*

1798, February 7, 11^h 7′. I cannot see the first satellite. The second is at a great angle sp near one of the stars marked down last night, which is now so small that I cannot distin-

* The planet being past the node, the angular distance of the satellites from zero, and their apparent motions are inverted.

guish it from the satellite. An extremely small star preceding a line that joins two lettered stars may be an exterior satellite. Position of the satellite (called the 6th) 64° 41′ np. 12^h 36′, there is no star in the path of the planet which to morrow or next day can be taken for this 6th satellite.

‡ The first satellite was 85° sp; distance 159, invisible. The second $78\frac{1}{3}$° sp; distance 592. The supposed satellite, called the 6th by way of readily referring to it, and also partly to express its distance, was found to remain in its observed place.

1798, February 11, 9^h 17′. The path of the planet is marked by a configuration of lettered stars, taking in those from which it comes and those towards which it will go.

11^h 35′, the situation of a supposed exterior satellite (called the 5th) with regard to the lettered stars is pointed out. It is excessively faint, but the night is very beautiful.

11^h 46′, the position of the 5th satellite is 89° 19,5 nf; but the satellite is so faint that the measure cannot be very accurate.

12^h 16′, I cannot see either of the two old satellites. There is an extremely small north preceding star *x*, which may be a more distant satellite; it is much smaller than the 5th, and will therefore become invisible when the planet comes near it.

‡ The first satellite was $89\frac{3}{4}$° nf; its distance was 41, and it was therefore invisible. The second was $79\frac{1}{2}$° nf; its distance was 241; and it was therefore also invisible.

1798, February 13, 10^h 17′. The old satellites are in the place I had calculated. The 5th satellite and the small star *x*, observed February 11, are not visible; but the weather is very indifferent.

11^h 49′, I do not see the 5th satellite where it was February 11.

12^h 0′, the position of the two old satellites is 76° 48′ nf.

12^h 15′, I see the extremely small star *x* remaining in its former place.

12^h 44′, the first and second satellites are exactly in a line pointing to the centre of the planet. A second measure of their position is 75° 45′. I cannot see the 5th satellite in the place where it was February 11.

‡ The first satellite was $78\frac{1}{4}$ nf; distance 594. The second was $78\frac{1}{2}$° nf; distance 576.

1798, February 15, 11^h 21′. There is a very small star in the line of the greatest northern elongation, it may possibly be an interior satellite, the first and second being invisible.

I see the extremely small star *x* of the 11th perfectly well, but the 5th satellite of the same night is gone from the place where it was that evening. It was considerably brighter than *x*, so that if it were in its place, I must certainly see it.

11^h 41′, the star which at 11^h 21′ I supposed might be an interior satellite is too far from the planet; it may possibly be the 5th satellite of the 11th on its return from the northern elongation towards the planet.

I believe there is another satellite or star between this last mentioned one and the planet; I do not suppose the second satellite to be visible, otherwise it would agree well enough with the situation of the star between the 5th and the planet. By the configuration the intermediate star is at about half the distance of the farthest of the two.

12^h 13′, position of the supposed 5th satellite 84° 49′ nf.

‡ The first satellite was 73° nf; distance 124, it was there-

fore invisible and could not be the supposed 5th satellite. The second was 77° nf; distance 455; it was therefore visible, and agrees very well with the satellite miscalled the 5th; the star between the second satellite and the planet must have been an interior satellite at its greatest northern elongation. At the time of observation, my defective tables made me suppose the nearest of the two to be the second satellite.

1798, February 16, 9^h 25′. The supposed fifth satellite observed last night at 11^h 41′, I believe is gone from the place where I saw it at that time. The night is very beautiful; the planet however is still low, and I shall look for it again when it is higher.

10^h 57′, the supposed fifth satellite is gone from its former place. It was so visible last night when near the planet, that I should certainly see it without difficulty if it remained in the same place, as the planet is now removed from it.

11^h 5′, the first and second satellites are invisible.

11^h 12′, there is a very faint satellite in the southern elongation; probably the sixth; and if it be the sixth satellite it is probably a little before or after its greatest elongation. It is excessively faint.

12^h 27′, the weather is not so clear now, though still fine, but the sixth satellite cannot be seen; it is plain, therefore, that the least haziness will render it invisible.

‡ The first satellite was $80\frac{1}{3}$° sp; distance 289, invisible; the second was $75\frac{1}{2}$° nf; distance 244, invisible. The interior satellite observed the fifteenth, being taken for the second, was not looked after; but as the supposed fifth was scrupulously ascertained to be removed, the interior satellite, had it been a star remaining in its former situation, must unavoidably

have been seen; for by the configuration they could not be much more than a diameter of the planet asunder.

1798, February 18, 9^h 19′. I see the sixth satellite observed February 16, at 11^h 12′, it has left the place where it was at that time. It is nearer the planet than it was that evening, I suppose it therefore to be on its return from its southern elongation.

There is a seventh satellite near the sixth, rather a little fainter than the sixth; a supposed eighth satellite is pointed out.

11^h 25′, the position of the sixth satellite is 80° 53′ sp.

11^h 31′, with 480 I see the satellite near the sixth perfectly well; the distance between the two is about $\frac{3}{4}$ or one diameter of the planet.

11^h 44′, I see the satellite much better with 600; that which is farthest from the planet is the largest.

‡ The first satellite was 78° sp; distance 563. This therefore might be the satellite which was seen near the sixth. The second satellite was $80\frac{1}{2}$ sp; distance 290, and was invisible. The supposed eighth, proved to be a star.

1798, February 19, 11^h 12′. The first satellite is invisible; the second is near the greatest elongation.

‡ The first was 81° sp; distance 269; invisible. The second was $79\frac{1}{2}$° sp; distance 490.

1798, February 22, 9^h 9′. The first satellite by its distance is not far from its greatest northern elongation; it is very large. There is a satellite to the south exactly opposite to the first; it is very small but may be the second. The moon is too bright to see very faint satellites.

‡ The first satellite was 78° nf; distance 594. The second

was 78° sp; distance 431. The small appearance of the second satellite is not easily to be accounted for; its distance from the planet was not much less than that of the first; for if the greatest elongations of the satellites be as 3 to 4, the above distance will be 1782 to 1724.

1798, February 26, 9^h 52′. The first and second satellites are in opposition; the first being sp, the second nf. The moon is so bright that their light is very feeble. Position 79° 53′ from sp to nf. The first satellite is small, the second is large.

‡ The first satellite was 78½° sp; distance 584. The second was 79° nf; distance 527. The different proportional light of the satellites in different situations, will lead us to suppose that they have a rotation on their axes. The twenty-second, when the second satellite was 78° sp, it was fainter than the first, and this evening when it was nf, it was brighter.

1798, March 11, 8^h 13′. With 300 the first and second satellites are close together, like a very faint double star of the first class. The second satellite is the most north of the two.

9^h 45, the position of the two satellites is 78° 15′,3 nf. There is hardly a division between them.

‡ The first satellite was 78¼° nf; distance 580. The second was 78¼° nf; distance 445; and supposing the diameter of the orbits of the two satellites to be as 3 to 4, their distances at the time of observation would be as 174 to 178.*

1798, March 12, 9^h 11′. The first satellite is nearly at the same distance from the planet as it was last night; the second

* The angular distance of the satellites from zero and their apparent motions are reverted.

is farther from the planet. The two satellites and the centre of the planet are exactly in a line. Their position is 78° 12',6. With 480 I had a glimpse of a south preceding satellite; but could not verify it with 600.

12h 6'. Distance of the second satellite 50",02. I contrived to throw a little light upon the wires, as the satellite was bright enough this evening to bear it.

‡ The first satellite was $78\frac{1}{4}$° nf; distance 528. The second was $78\frac{1}{4}$° nf; distance 577.

1798, March 13, 11° 46'. The first satellite is invisible. The second is much nearer the planet than it was last night. The weather is not clear, owing to easterly winds.

‡ The first satellite was $78\frac{1}{4}$° nf; distance 170; invisible. The second was $78\frac{1}{4}$° nf; distance 577.

1798, March 14, 11h 55'. The first satellite is invisible; 8h 31', the second is still at a considerable distance south preceding.

11h 47', twenty-five reflector, power 200. The Georgian planet is better defined in this instrument than I have ever seen it before. With 300, its disk is as sharp and well defined as that of Jupiter. The second satellite is brought to a sharp point. A little while ago I had a glimpse of a south preceding satellite, and just now I have seen it again. 12h 0', I cannot verify the satellite, but can hardly believe it a deception.

Twenty feet reflector, power 300. I tried to measure the distance of the second satellite, but its present faintness will not afford light enough to see the wires of the micrometer.

‡ The first satellite was $78\frac{1}{4}$° sp; distance 260, invisible; but the 25 feet telescope with a mirror of 24 inches in

diameter, it appears, had light enough to show it. The second satellite was $78\frac{1}{4}°$ nf; distance 473.

1798, March 16, 8^h 37′. I see a south preceding satellite at a good distance; it may be the first at its greatest elongation, but it is certainly smaller than it should be; unless the state of the air should be worse for seeing than it appears to be.

9^h 31′, the distance of the first satellite is 36″,05. The satellite is so faint that it is impossible to be very accurate; it will not bear any light to the wires.

11^h 34′, twenty-five feet reflector. With 300 I see the satellite very distinctly, but the evening is not fine.

‡ The first satellite was $78\frac{1}{4}°$ sp; distance 576. The second at 8^h 28′ was $78\frac{1}{4}°$ sp; distance 13; invisible.

1798, March 18, 8^h 26′. The first satellite is invisible; the second is south preceding at a considerable distance; it is farther off than the greatest elongation of the first.

‡ The first satellite was $78\frac{1}{4}°$ nf; distance 51; invisible. The second was $78\frac{1}{4}°$ sp; distance 484.

1798, March 19, 9^h 51′. I see a north following satellite which I suppose to be the first. The second is near its south preceding greatest elongation. Distance 49″,90, I can only apply a very distant lantern, which will hardly give light enough to show the wires. The satellite is not so bright in its southern elongation as it was March 12th in its northern one, though the weather is now very beautiful. In the south preceding elongation is a distant star that may be a satellite; many other stars are pointed out.

‡ The first satellite was $78\frac{1}{4}°$ nf; distance 445. The

second was $78\frac{1}{4}°$ sp; distance 588. The stars remained in their places.

1798, March 21, 10^h 25′. The first satellite is north following; it is faint, and at nearly the same distance from the planet as it was in March 19. The second satellite is near one of the stars pointed out the 19th, both being at the same distance from the planet. 10^h 40′, there is such a multitude of small stars in the neighbourhood of the planet, that it would be endless to look for the additional satellites among them.

‡ The first satellite was $78\frac{1}{4}°$ nf; distance 438. The second was $78\frac{1}{4}°$ sp; distance 409.

1798, March 22, 10^h 35′. The first and second satellites are both invisible.

‡ The first was $78\frac{1}{4}°$ nf; distance 58; invisible. The second was $78\frac{1}{4}°$ sp; distance 173; invisible.

1798, April 6, 8^h 31′. The first satellite is north following; I suspect the second to be between the first and the planet, but cannot verify the suspicion. There is a supposed south preceding satellite, but it is too near the planet to be seen steadily.

‡ The first satellite was 78° nf; distance 564. The second was $75\frac{1}{3}°$ nf: distance 230; it was therefore the satellite suspected between the first and the planet. The supposed south preceding satellite was lost among the numerous small stars.

1798, April 7, 9^h 26′. There are two satellites north following; they are very near together. The distance between them is less than half the diameter of the planet. The centre of the planet and the two satellites are exactly in a line.

There are so many small stars that it is next to impossible to look for the additional satellites.

‡ The first satellite was 79° nf; distance 546. The second at 8^h 45′ was $77\frac{1}{3}$° nf; distance 453.

1798, April 8, 10^h 19′. There is no satellite visible between the second and the planet; the second satellite is north following, at a greater distance from the planet than last night. There is a very small star at a little more than twice the distance of the second satellite north following.

‡ The first satellite was $80\frac{1}{2}$° nf; distance 239; invisible. The second was 78° nf; distance 576. The small star remained in its place.

1798, April 9, 9^h 34′. I cannot see the first satellite. The second is at a distance north following, rather farther from the planet than last night. With 480 and 600, there is no satellite between the second and the planet.

‡ The first was $74\frac{1}{2}$° sp; distance 171; invisible. The second was 79° nf; distance 578.

1798, April 11, 9^h 8′. The first satellite is south preceding at a considerable distance; but not at its greatest elongation. I cannot see the second satellite. I suspect a very small star in the line of the north following greatest elongation, a little farther from the planet than the first satellite. With 480 I cannot verify the suspicion.

‡ The first satellite was $78\frac{1}{2}$° sp; distance 587. The second was $80\frac{1}{4}$° nf; distance 247; invisible. This satellite could hardly be the suspected star, as it was but at little more than half the distance of the first satellite from the planet.

1798, April 12, 9^h 54′. I cannot see the first satellite, nor the second. With 480 there is no satellite either new or old visible. The night seems to be very clear, but the wind is in the north-east.

‡ The first satellite was $80\frac{2}{3}$° sp; its distance was 382, and it ought to have been seen. The second was 61° sp; distance 46; invisible; if the north-east wind, which is always unfavourable for astronomical observations, prevented my seeing the first satellite, it could not be expected that the suspected star of the eleventh would be visible.

1798, April 13, $9^h\ 5'$. The first satellite was not seen. There is a south preceding satellite at a little greater distance than half the greatest elongation of the first. It took some time to verify its existence. 400 shows it very well; it is the second satellite. The very small stars in this neighbourhood are so numerous, that it is impossible to look for the new satellites among them. A great many of the stars are pointed out.

‡ The first was 1° sf; distance 16; invisible. The distance being so small, the method here used is not sufficient to give an accurate position. The second satellite was 76° sp; distance 301.

1798, May 3, $10^h\ 3'$. The first satellite is north following. $10^h\ 7'$, I suspect another north following a little nearer the planet than the first; 460 almost verifies it.

‡ The first satellite was $79\frac{1}{3}$° nf; distance 556. The second, which was the suspected one, was $74\frac{1}{2}$° nf; distance 259; it was therefore seen at a distance considerably less than half the greatest elongation.

1799, April 3, $9^h\ 51'$. I viewed the Georgian planet with 157 and 300; one of the satellites is about 10° sp.

‡ The first satellite was 77° sp; distance 581; the second

was 66° sp; distance 172; this being invisible and the first at so great an angle, a star must have been taken for a satellite, which might well happen after an interval of eleven months. The observation was chiefly made to try two newly polished mirrors.*

1799, April 8, $9^h\ 51'$. Both the old satellites are in a line near the greatest north following elongation. The nearest is very faint.

‡ The first satellite was $74\frac{1}{3}$° nf; distance 452; it was the faint satellite and the nearest of the two. The second was $75\frac{3}{4}$° nf; distance 529.

1800, March 26, $11^h\ 0'$. With a new mirror and power 300 I saw the planet beautifully well defined, and one of its satellites south preceding.

‡ The first was $74\frac{3}{4}$° sp; distance 566, it was therefore the observed satellite. The second was $31\frac{2}{3}$° nf; distance 134; invisible.

1800, April 26, $9^h\ 30'$. I see two satellites almost in opposition; the south preceding one is the largest and at the greatest distance. I see also several extremely small stars; but without a regular succession of observations, it is impossible to determine whether any of them may be satellites.

‡ The first satellite was 71° nf; distance 469. The second was $76\frac{1}{3}$° sp; distance 592. They were the observed satellites, and the apparently inverted direction of their motion is evident.

1801, March 8, $12^h\ 0'$. The first satellite is at a great angle south preceding; the second at a great angle north

* The planet being now again past the node, the angular distance of the satellites from zero, and their apparent motions are again inverted, and will remain so.

following; but a line from the second drawn through the planet, leaves the first satellite on the following side.

‡ The first was $85\frac{3}{4}$° sp; distance 533. The second was $78\frac{1}{4}$° nf; distance 599.

1801, April 17, 10^h 30′. The first and second satellites are in view at great angles north following the planet. There is a third satellite at a great angle south preceding; in the configuration it is marked exactly in opposition to the second, and at half the distance of the first. Six stars are pointed out.

‡ The first satellite was $77\frac{1}{4}$° nf; distance 598. The second was 81° nf; distance 586; and the third by the configuration was 81° sp.

1801, April 18, 10° 26′. The first and second satellites are in the configuration at great angles north following. The six stars of last night are in their places, but I do not see any star where the third satellite was marked.

‡ The first satellite was 65° nf; distance 438. The second was 74° nf; distance 578. The third was probably the interior satellite at its greatest southern elongation, which cannot be visible two days together. The configuration of this evening, compared with that of the night before, shows by the situation of the satellites, that their apparent motion is in an inverted direction.

1801, April 19, 10^h 24′. The first satellite was not observed. I saw the second satellite advanced in its orbit in the inverted order. The moon is too bright to make observations on additional satellites.

‡ The first was $7\frac{1}{2}$° nf; distance 144; invisible. The second was 66° nf; distance 438.

1808, May 27, 10^h 0′. The planet is too low to admit the use of very high magnifying powers. With 300, however, I have a glimpse of what I suppose to be the two large satellites. A haziness coming on will not permit the angle of position to be taken.

‡ The first was 47° sf: distance 478; the second was 56° np; distance 509. They were therefore both visible.

1809, May 12, 11^h 0′. I viewed the planet with 300, but could not perceive the satellites. The planet is too low, and there is a strong twilight.

‡ The first satellite was 72½° sp; distance 578. The second was 53° np; distance 529.

1810, May 25, 10^h 40′. I viewed the Georgian planet with the 40 feet telescope, power 400. The disk of it is very bright. Several small stars are near it, but without a series of observations, it cannot be possible to ascertain which of them are satellites. What I suppose to be the second is 65° or 70° nf.

‡ The first was 87¼° np; distance 594. The second was 76° np; distance 588. Both satellites were therefore visible, but being among surrounding stars, could not be distinguished from them.

Investigation of several particulars deduced from the foregoing observations, with an exposition of the method by which they have been obtained.

The first use to be made of the numerous angles of position that have been taken, must be an investigation of the place of the node, and the inclination of the orbits of the satellites. When these two particulars are obtained, the times of the periodical revolutions of the satellites about the planet, may be settled. It will then be necessary to calculate the places of the satellites for the times in which they were observed, in order to identify them; for, notwithstanding all possible care was taken to keep them in view, yet after the long unavoidable annual interruptions, and the periodical interference of the moon, it will be seen, that several mistakes have been made in naming the satellites, which by that means may be easily corrected.

The place of the ascending node, the inclination of the orbits, and the retrograde motions of the satellites determined.

When the observations of the satellites in the year 1797, and the beginning of 1798, are examined, it will be found that the first satellite could seldom be seen, and that its positions, when observed, were always at a great angle from the parallel; the second was also frequently invisible; and its observed positions were likewise at great angles. From these appearances it may be concluded that the planet was approaching to the node of the satellite's orbits. At the latter end of February, and the beginning of March 1798, the position of both the satellites approached to a settled angle, which at last, for two successive days, namely the 11th and 12th of March, remained stationary; I have therefore supposed that the planet was then in its passage through the node, and have in my calculations admitted its place to be five signs, 15 degrees, 30 minutes.

Then a mean of the angles of position 78° 15',3 and 78° 12',9 taken the 11th and 12th of March, being 78° 14' north following, it will appear from the method of calculation which will be explained, that the inclination of the orbits of the satellites to the ecliptic is 78° 58′.

From these data it also follows, that the motion of the satellites in their revolutions round the planet, by which they are carried from their ascending node to their greatest elongation, is retrograde.

Consideration of the principles by which the periodical revolution of the satellites may be obtained from the observed angles of position.

It will be necessary to premise, that, in order to simplify the investigation of the periodical revolution of the satellites, I have supposed the orbit of the Georgian planet, which differs only about $\frac{3}{4}$ of a degree from the ecliptic, to be coincident with it.

When the rate of the motion of a satellite in its orbit is to be determined from two observed situations, it is required to reduce its first apparent place on the plane of projection, to its real situation in its orbit, I have therefore taken the ascending node for a fixed point, from which we may begin to number the degrees of the satellite's situation; then, as in the second observation, the satellite must also be brought from its apparent place to its real one, we have to allow for three material alterations, that will more or less affect the calculation, according to the length of the interval of time between the two observations.

The first of these alterations is that which takes place in the situation of the parallel, from which the angles are measured; the second is a change in the inclination of the plane of the orbit of the satellite to the plane of projection; and the third is an alteration in the distance between the ascending node and the extreme point of the transverse axis of the ellipsis, into which the orbit is projected.

In figure 1, (Pl. XVI.) suppose the circle PSFN to represent the plane of the orthographic projection, in which the angles of position are counted from the parallel P and F towards S and N, and expressed by the number of degrees 10, 20, 30 to 90 in each quadrant. Then let there be a moveable circle within the former, of which the degrees should be marked in succession, 10, 20, 30 continued to 360. The inner circle being moveable, the line from 180 to 360 will express, by its different situations, the position of the transverse axis of the ellipsis into which the orbits of the satellites at any given time are projected. The conjugate extending from 90 to 270, and the lines parallel to it, will point out the direction in which the orbits will be more or less contracted according to the different inclinations of their planes to the plane of the projection.

In a triangle PS*p*, figure 2, let P be the pole of the ecliptic. S, a point in the orbit of the satellite, 90 degrees distant from the ascending node. *p*, the point of the greatest northern elongation of the satellites, which on the moveable circle is marked 360. It is the zero of the ellipsis into which the orbits of the satellites are projected, and its calculated situation will regulate the adjustment of the moveable circle. N, a point in a meridional direction, 90 degrees distant from the geocentric place of the planet. The arch PS then being the complement of the inclination of the satellite's orbit to the ecliptic, is therefore given; and the angle at P is equal to the distance of the longitude of the planet from the node. The angle at S is a right one; and the arch PN, being the measure of the angle of position of that point of the ecliptic where the planet is situated, may be had by Table LIII, published in Dr. Maskelyne's first volume of Observations.

To find what alteration has taken place in the situation of the parallel with regard to the point *p*, we calculate the position of this point for any given time by the following analogy. (1) cos P : rad : : tan PS : tan Pp; and the difference between PN and Pp will give Np; the complement of which is the position of *p*, with regard to the parallel. When P*p* is greater than PN, the position of P being north following, that of *p* will be north preceding; but when P*p* is less than PN, it will be north following.

To find the inclination of the plane of the satellite's orbit to the plane of projection, we have only to calculate the distance of the poles of these planes; and since the place of the planet is the pole of the projecting plane, and since also the situation of the pole of the orbits is known, which is in longitude 15 degrees 30 minutes of Gemini, and latitude south 11° 2′, therefore in the right angled triangle figure 3, of which GL is the distance of the planet from the longitude of the pole, and PL the given latitude of the pole, we have the analogy (2) rad : cos GL : : cos PL : cos GP; which is the required inclination of the two planes to each other.

To find the distance of the point *p*, or zero of the projected ellipsis from the ascending node, by figure 2 we have the analogy (3) rad : sin PS : : tan P : tan S*p*; and 90° + S*p*, before the node, and 90°—S*p*, after it, will give the distance of *p*, or zero point of the inner circle from the ascending node.

The periodical revolutions of the satellites determined.

In the natural order of investigating the motions of the satellites, the first consideration ought to be to identify the observations, lest a star should have been mistaken for a satellite, or one satellite for another; but as the calculations required for this purpose cannot be made without proper tables of their periodical revolutions, I have proceeded in the following manner.

The earliest angles of the positions of the satellites which appeared to be sufficiently accurate for the purpose of settling their motions were taken 1787, February 16, 9^h 38′ mean time. With a wire for the parallel in the focus of the eye-glass, and a magnifying power of 300, the position of the first satellite was five degrees north following; and that of the second was three degrees south following: the motion of the satellites being so near the parallel, there can be no material error in the estimation of the angles; and to prevent the influence of a diversity of errors, I have fixed upon the above-mentioned time as a general epoch to which every calculation of the motion of the satellites has been referred, not only in the determination of the periods, but also in every identification.

With the assistance of the analogies that have been given, seven single periods of the revolution of the first satellite were calculated, from the union of which a general compound period has been deduced. The single periods were calculated from a combination of the observation of the 16th of February 1787, with one of the same year, and with six more of the years 1790, 91, 92, 93, 94 and 96.

In the same manner, nine single periods of the second satellite have been calculated by combining the observation of the 16th of February 1787, with one or more of the succeeding, as far as 1797: the observations of 1798 being too close to the node to give a result that might be depended upon.

It will be proper to give the particulars of one of the single periods, to show what degree of accuracy has been used in the calculation.

By observation, the position of the second satellite, February 16, 9^h 38′ 1787, was 3° sf. The situation of the parallel of declination, by analogy (1) was such that the point F, upon the outer circle of Fig. 1. was opposite to 261° 14′ of the inner circle; 3° sf therefore, was at 258° 14′ of the inner circle. By analogy (2), we then find the inclination of the plane of the orbit to the plane of projection, which enables us, by the argument of the satellites being 78° 14′ from the greatest southern elongation, to reduce the apparent place in the circle to the real one, in the orbit elliptically projected. The correction for this reduction will be + 2° 27′; which being applied, gives 260° 41′. But this being the situation which is numbered from the moveable zero, marked 360, it must be brought to its fixed distance from the ascending node by analogy (3); which gives the distance of the zero from that node for this day 104° 25′; and this being added to the former quantity, gives the real place of the satellite in its orbit from the ascending node 5° 6′.

To combine this with the observation of March 28, 10^h 36′ 1797, when the same satellite was 80° 44′ nf; we find that F, in the parallel should now point to 281° 4′ of the inner circle; and that consequently 80° 44′ on the outer circle, will be opposite to 1° 48′ of the inner circle; and that to reduce it by the inclination of the orbit, a correction of + 15° 41′ must be applied, which gives its situation in the apparent elliptical orbit 17° 41′ from zero. And when the distance of this point from the ascending node, which now is 91° 6′, is added, we have the satellite's real place in its orbit 108° 47′. Then as in 1787, it was at 5° 6′, and is now at 108° 47′, it must have moved over an arch of 103° 41′ of its orbit, to which, if we add 274 revolutions, we find that the sum of its motion amounts to 98743,68 degrees. The interval of time, in which it has moved over this number of degrees, will be found to be 3693,040277 .. days; from this we obtain the required periodical time, which is $13^d\ 11^h\ 8'\ 19''$. This single period differs only 40″ from the compound mean period of the revolution of the second satellite.

The seven detached periods of the first satellite, and the nine of the second have all been calculated in the same manner; and in order to obtain a mean value of them, I judged it proper to allow to the duration of every interval of the time, for which they were calculated, its due weight in the scale, by compounding them together. This was done by adding together the single intervals of time in each period, and also adding together the number of degrees passed over in each single period, and computing then the compound period by these collected sums of times and motions, the result of which is, that the first satellite makes a synodical revolution about the planet in $8^d\ 16^h\ 56'\ 5'',2$, and the second in $13^d\ 11^h\ 8'\ 59''$.

Explanation of the identifying method.

It is evident, that we cannot be satisfied with a conclusion that is drawn from the apparent situation of an observed satellite, if a doubt should remain whether it actually was the satellite which it is said to be; and where such numerous observations are to be examined, a method of identifying the satellites becomes absolutely necessary.

When the periodical revolution is known, the place where a satellite at any given time should be seen, may be strictly calculated; but a method somewhat less rigorous, and much more expedient, will be sufficient for the purpose; but even this will be found to require tedious computations; for in the first place, the motion of the satellite to be identified for any day, must be cast up by the table of its motion in days, hours, and minutes; and for this purpose, the interval of time for which it is calculated, must first be ascertained; this has been done for every day the satellites have been observed. The amount of the motion in orbit being obtained, it must be added to the number of degrees from which the motion proceeded; this at the already mentioned general epoch of 1787, Feb. 16, 9° 38′, was for the first satellite 11° 27′ from the node, and for the second 5° 6′. The sum then will be the real place of the satellite in its orbit.

Now, to obtain the apparent place of a satellite from a given real one, a table must be made, the first column of which must contain the degree of the geocentric longitude of the planet, for which the rest of the columns are calculated. The three analogies that have been given, are to be used for obtaining the contents of the second, third, and fourth columns;

the fifth, contains the natural sine of the inclination given in the fourth column multiplied by 6.

Such a Table has also been calculated for every degree, from three signs 20° to seven signs 12°, which takes in the whole compass of the observations that have been given. I insert the three first lines of the Table as a sample of its construction.

Geor. longitude of the planet.	Position of 360 or zero.	Distance of zero from ditto.	Inclin. of the plane of the orbits to the plane of projection.	Natural sine of the Inclin. × 6.
3^s 20°	79° 27′ np	105° 34′	36° 1′	4,85
21	80 17	105 1	36 57	4,79
22	81 5	104 30	37 54	4,73

In order now to use these preparatory calculations, I made an apparatus consisting of a square piece of pasteboard, upon which a circle was drawn and graduated as in figure 1. To the centre of this, I joined a moveable circle also drawn upon pasteboard, and graduated as in the figure. The radius of the inner circle was exactly six inches, and its circumference was nearly in contact with the inside of the outer circle. From what has already been said of the construction of this figure, its use in the identification of the satellites, will easily be understood by a few examples of it.

With the geocentric longitude of the planet taken from the Nautical Almanack, I take out the required quantities from the different columns of the Table. In this operation it might be sufficient to take only the nearest degree for entering the

Table; but as the difference between any two degrees may be had by inspection, I have always used the nearest half degree. For instance, the quantities for the half degree between three signs 21° and 22°, in the second, third, and fifth columns, will be 80° 41′; 104° 45′; and 4,76. And the same quantities will do for any day from March 7, 1787, till April 23, for which day the quantities must be taken from three signs 22°, &c. &c.

Now, suppose it be required to ascertain whether a satellite called the second, which the 15th of March 1787, at 8^h 7′ was observed to be five degrees south following the planet, was indeed the second satellite? Then I see in the general list of calculated motions, that from February 16, 9^h 38′ to March 15, 8^h 7′, is an interval of 26^d 22^h 29′; in which the second satellite has moved 0° 12′ from its place; and as it was then at 5° 6′, it is therefore now 5° 18′ from the ascending node of its orbit.

In using the identifying apparatus, the first thing to be done is, the adjustment of the inner circle to the position it ought to have for the day of observation, which is pointed out by the geocentric longitude of the planet three signs $21\frac{1}{2}$ degrees; the zero must consequently be adjusted to 80° 41′ north preceding; I therefore turn the inner circle upon its centre, till the point 360 is opposite to $80\frac{2}{3}$° np; for in the adjustment of the circles to each other, and in reading off the angles pointed out by them, a critical estimation of minutes has not been attempted; whenever, therefore, minutes are given, they must be understood to relate to calculations, or to measures taken with a micrometer.

In the next place, the point of the inner circle, answering to the calculated situation of the satellite in its orbit, is to be found by the tabular quantity 104° 45′, which is the distance of the ascending node from the zero of that circle; and as the satellite is 5° 18′ from the same node, the quantity given by the Table must be deducted from the same; that is from 365° 18′, and the remainder 260° 33′ will be the place of the satellite on the moveable circle.

Finally, to get the angle of position at which the satellite will be seen, the two ends of a proportional compass must be adjusted to each other, so that when one end of it is opened to six inches, the other may give the quantity in the last column of the Table, which in the present case is 4,76 inches. The distance from $260\frac{1}{2}$° to the transverse must then be measured by the long end of the compass, in a direction parallel to the lines in the figure; and the opening of the short end must be set back again in the same direction, from the transverse towards $260\frac{1}{2}$°; a fine point must be marked with the end of it upon the pasteboard. A black silk thread fixed to the centre of the circle, may then be stretched over the impressed point to intersect the degrees of the outer circle, upon which the positions are reckoned in the order they are marked; and this being done, it will be seen that the intersection of the thread falls upon $2\frac{1}{4}$ degrees of the south following quadrant; which sufficiently identifies the satellite.

In observations that are made when we are in, or very near the node of the orbits of the satellites, their angular positions undergo hardly any change, and can therefore be of no use for identifying them; but they may then be distinguished by their proportional distances from the planet; and these may be very conveniently had in six hundredth parts of the respective radii of the satellites, and the impression of the fine point whereby the angle of position is obtained will be of eminent use; for by putting one leg of the compass upon this point, and extending the other to the centre of the circle, we shall in the present case have 4,81 inches for the measure of the required distance, which as the radius of the circle is six inches, will be $\frac{481}{600}$ parts of it; and in such parts all the distances which are given in the foregoing observations have been expressed.

I have called this manner of obtaining the angles of position and proportional distances, the identifying method, that it may remain distinguished from strict computation; there is, however, so much real calculation mixed with it, that I may confidently draw the following interesting conclusions from it.

I. With the light of my 20 feet telescope, the first satellite generally becomes invisible at the distance of a little more than half its greatest elongation; I suppose it to be when the identified measure of it is from 302 to about 310.

II. The second satellite becomes invisible at very nearly half the distance of its greatest elongation; I suppose it to be when its identified distance is from 295 to about 305.

III. An interior satellite as large as the first, must be more than half the greatest elongation of the first satellite from the planet; and if it be smaller, it must be at so much greater a distance from the planet, to be seen at its greatest elongation. Nor can there be any chance for seeing it two nights together, when the orbits are contracted by projection.

IV. Exterior satellites that are very faint when at their greatest elongation, can hardly ever be seen at any other time when the orbits are contracted.

V. The first satellite is probably larger than the second; for though the latter is generally the brightest, it seems to be only in consequence of its being farthest from the planet. On comparing the limit of its disappearance with the number 302, expressing that at which the first satellite generally ceases to be visible, we find that the second satellite, upon its own scale, should not be lost in the light of the planet till it came within the limit of 224, instead of 295.

VI. Both the satellites are subject to great variations of light, not owing to the changeable clearness of the air at different times; for by comparing the brightness of one satellite with that of the other when they are seen together, the state of the air will be of equal clearness to both, and yet their comparative brightness has been observed to be very different: for instance, March 14, 1793, the first satellite was brighter than the second, when the distance of the former was to that of the latter as 172 to 235; and February 26, 1798, the first was small, and the second larger when the distance of the former was to that of the latter as 175 to 210.

VII. The variable brightness of the satellites may be owing to a rotation upon their axes, whereby they alternately present different parts of their surfaces to our view. These variations may also arise from their having atmospheres that occasionally hide or expose the dark surface of their bodies, as is the case with the sun, Jupiter and Saturn.

VIII. The real angular distances of the satellites from the planet may be determined from the measures that are given in the observations, but to enter critically into this subject would extend far beyond the compass of this Paper. The disagreement of the measures is very considerable; this will, however, not appear so remarkable, when the faintness of the satellites is considered, which will not admit of an illumination of the wires of the micrometer. The two measures of the distance of the first satellite that were taken Oct. 11, 1787, and March 12, 1798, should both be considered; but a selection of about six of the most consistent measures of the second satellite, will probably be necessary to give the truest result. For instance, those of 1787, March 18, 20, April 9, and November 9, with those of 1798, March 12 and 19, might be taken. If these measures are brought to their greatest elongation by the identified distances that are given with them, some kind of judgment may be formed of the probable result, when calculation is applied to them.

In my observations I have supposed the distances of the first and second satellites to be 36 and 48 seconds, and by this proportion I have occasionally reduced the identified distances of the two satellites to an equal scale.

IX. The existence of additional satellites has already been considered in a former paper.* Many remarks on them were given under the four heads of interior, intermediate, exterior, and more distant satellites; and, as many additions are contained in the foregoing observations, I shall review the former remarks, with the assistance of the light which the identifying method has thrown upon them, and afterwards, in the same order consider, in each class, what evidence of the existence of such satellites may be derived from the additional observations, especially from those that were made in the year

* Phil. Trans. for 1798, page 59.

1798, when the orbits of the satellites were contracted into a line, which might be examined with greater facility than a more expanded space; and where even the very situation of a star in this given direction, rather than in the numberless others, in which it might be placed, must be a presumption of its being a satellite, provided its distance at the same time should not exceed a certain probable limit.

An interior satellite.

The supposed interior satellite observed January 18, 1790, could not be a star, whose existence was doubtful, as it had light enough for an estimation of its distance in diameters of the planet; its absence, however, not having been noticed the 19th of January, although great attention has been shown to ascertain the sidereal nature of another supposed satellite, observed at the same time with the former, leaves the observation of the eighteenth unsupported.

The observation of the 4th of March 1794, which has been supposed to relate to an interior satellite, is by the identifying method proved to belong to the second satellite; and its observed absence on the 5th from the place where it was the 4th, being thus verified and accounted for, shows that great confidence may be placed on such observations.

The observation of an interior satellite of the 27th of March, 1794, is without a subsequent observation; but then it has already been noticed in remark III, that an interior satellite cannot be seen two successive days, when its orbit is already contracted, as it was on the day of observation.

Addition.

The 15th of February, 1798, an interior satellite was seen about its greatest northern elongation; as it was between the planet and the second satellite (miscalled the fifth), its position must have been 84° 49′ nf. On account of its faintness it was not seen immediately, but as soon as it was perceived, it was surmised to be the second satellite; but the identified distance of the second, which was 455, is much too far for the observed distance of the faint satellite, and proves that in reality the supposed fifth was the second satellite. This is moreover confirmed by its brightness, and by the angle of position which was taken. The first satellite was invisible, its distance being only 124; it even remained invisible the next day, when its distance was 289; the observed faint one, therefore, must have been an interior satellite in a distant part of its northern elongation. The 16th of February, the place where the satellite had been the day before was scrupulously examined in looking for the supposed fifth, and as there was no star remaining in that place, the removal of the interior satellite from its former situation was thereby also ascertained. It has already been noticed that in the contracted position of the orbits an interior satellite observed the 15th could not possibly be seen the 16th, which accounts for its not being noticed the last day of observation. This is one of the cases where the singular situation of a star alone, is almost sufficient to prove it to be a satellite.

The 17th of April, 1801, the interior satellite, which had been seen in its greatest northern elongation, was now seen about its greatest southern elongation. Its situation, by identification, was 81° sp. The 18th of April, when the stars that were pointed out were looked after, and were all found remaining in their places, no star could be seen where the interior satellite had been situated the 17th; nor could it be expected to be visible, as, by its motion towards the planet, it must already have been involved in the splendour of its light.

An intermediate satellite.

The 26th of March, 1794, an intermediate satellite was seen; by the configuration its distance was greater than that of the first, but less than that of the second. By identification its situation must have been $59\frac{1}{2}°$ nf. The 27th of March, the satellite was no longer in the place where it had been seen the 26th, and moreover, a very small star was seen in a place that agreed with what would be the situation of an intermediate satellite, had it accompanied the planet.

An exterior satellite.

The 9th of February, 1790, an exterior satellite was observed. It was by the configuration at double the distance of the second satellite, and by identification, its position was $61\frac{1}{2}°$ sf. The observations of two succeeding days proved, that it remained no longer in the place where it had been seen on the 9th.

The 27th of March, 1794, some distant stars south of the planet were observed as being supposed satellites; but they are not sufficiently supported by succeeding observations.

The 5th of March 1796, a star was seen, which the night before had not been in the place where it was at the time of the observation. By the configuration its identified place must have been 72° np; and its distance exceeded that of the second satellite.

Addition.

The 31st of January, 1791, a satellite in opposition to the second, and at about double the distance from the planet was observed; its identified position was $78\frac{1}{2}$° np. The 2d of February all the stars of the configuration that had been pointed out, were seen remaining in their places, but the exterior satellite was not among them.

The 26th of February, 1792, a star at double the distance of the first satellite was pointed out, but it has not been accounted for in succeeding observations. By remark IV, however, faint exterior satellites can hardly be expected to be seen at any other time, than when they are about their greatest elongation.

The 11th of February, 1798, an exterior satellite, called the fifth, was observed; its situation was measured 89° 19′,5 nf. The 13th of February, this satellite was no longer in the place where it had been seen the 11th. The 15th of February it was again ascertained that the satellite had left its former situation. The orbits of the satellites, at the time of these observations, were already contracted into a line, and a very faint satellite like this could not remain visible two, and four days successively; its motion, according to remark IV, would in a short time immerge it again into the effusive light of the planet, and render it invisible.

More distant satellites.

The 28th of February, 1794, a small star was seen in a place where the 26th there was none. By the configuration of that day, and the identifying method, it was at a considerable distance, about 24 degrees north following the planet, and not far from a lettered star which was smaller than the new star. It cannot be supposed that a larger star should have been omitted to have been marked in the situation pointed out by smaller lettered stars, where it must have been seen the 26th, if it had been there.

The 27th of March, 1794, south of the planet, at a considerable distance, were small stars, that had the appearance of satellites; but there are no subsequent observations of them.

The 28th of March, 1797, a distant star is mentioned that was not seen the 25th, although the situation of the lettered stars of that day was carefully examined.

Addition.

The 16th of February, 1798, at $11^h\ 12'$ a very faint satellite, called the sixth, was observed, and from its distance, supposed to be a little before or after its greatest southern elongation. It was so faint, that a small alteration in the clearness of the air, rendered it invisible. On the 18th the sixth satellite was seen again, and, being nearer the planet than it was on the 16th at $11^h\ 12'$, it was supposed to be on its return from the greatest southern elongation. It was also ascertained on the 18th, that it had left the place where it was seen on the 16th. The angle of its position, by a measure taken of it, was $82^\circ\ 55'$ south preceding.

Fig. 1.

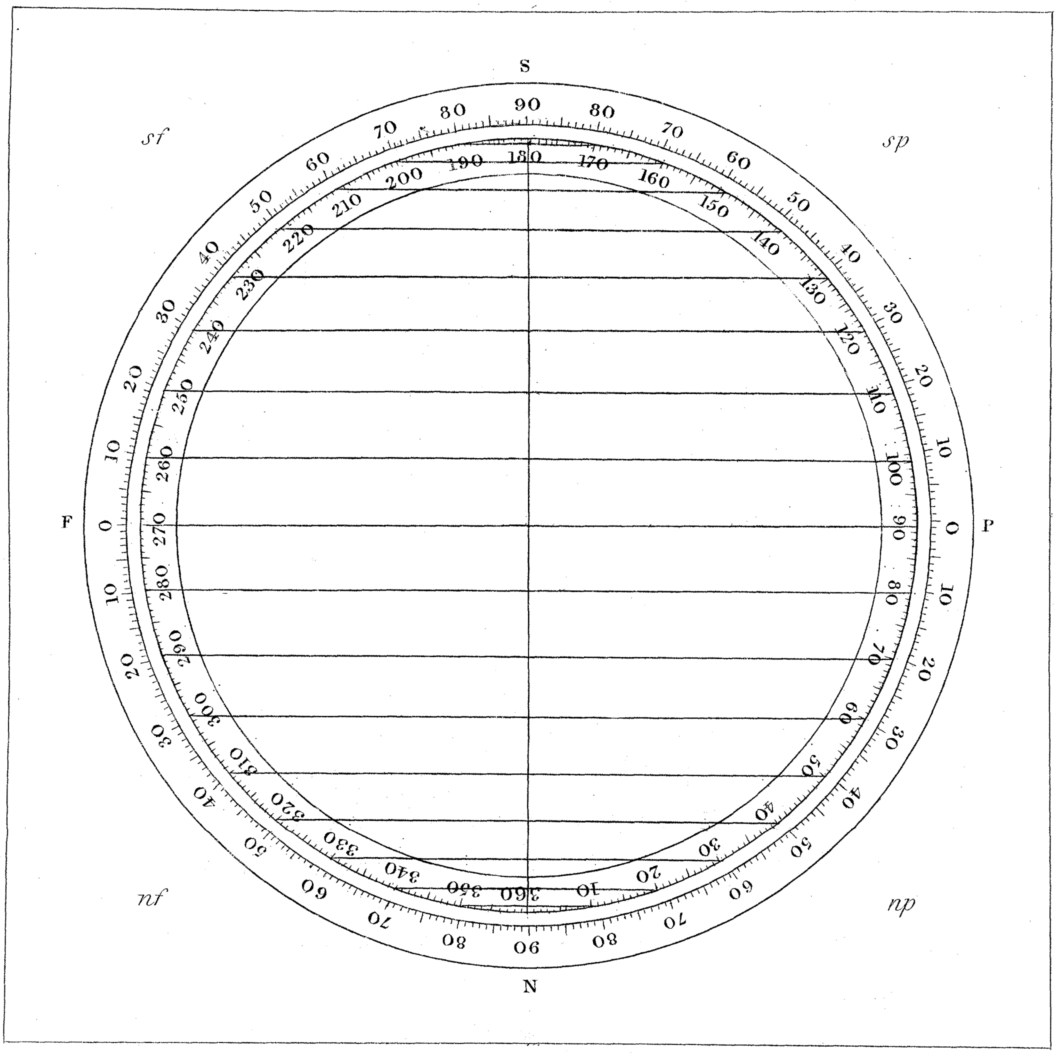


Fig. 2.

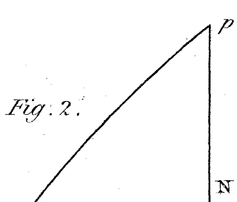
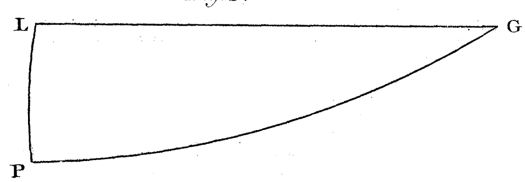


Fig. 3.



Jas. Basire, sculp.